Nonlinear Fourier Analysis of Ocean Waves:
Rogue Wave Dynamics in Random Wave Trains

A. R. Osborne
Dipartimento di Fisica Generale, Università di Torino
Via Pietro Giuria 1
10125 Torino, Italy
Phone: (+39) 11-670-7451 or (+39) 11-813-4462 Fax: (39) 11-658444
E-mail: al.osborne@flashnet.it

Grant #: N00014-98-1-0192

LONG-TERM GOALS

Application of the *inverse scattering transform* (IST) to the *time series analysis of laboratory and oceanic wave data*. The approach may be viewed as a generalization of linear Fourier analysis and is loosely referred to as "Nonlinear Fourier Analysis or Generalized Fourier Analysis." The goal is to develop time series analysis methods that allow for the processing of a wide variety of surface wave data.

A major focus in the last two years has been the application of IST to the study of *"rogue, freak or giant" ocean waves*. The present report emphasizes new results that have arisen from the study of numerical simulations of *random ocean wave trains*. The emphasis has been on the study of the *physical mechanisms leading to the generation of rogue waves in random sea states*.

OBJECTIVES

1. The long-term goal of the present research program is the development of *fast numerical inverse scattering* techniques (FIST) to enable the nonlinear Fourier analysis of nonlinear ocean wave data and to consequently improve our understanding and predictive capability of ocean surface waves.

2. The main objective during the past year has been the *numerical study of nonlinear random wave dynamics* and the *statistical study of the rogue waves* that may occur in these random wave trains. Additional studies emphasize the development of new IST algorithms that allow rogue wave dynamics to be studied as an exercise in nonlinear Fourier analysis.

APPROACH

There exist large classes of nonlinear partial differential wave equations that are completely solvable by the inverse scattering transform. In particular the Korteweg-deVries equation (unidirectional shallow water wave motion) and Kodomtsev-Petviashvilli equation (two-dimensional shallow water wave motion) are being studied. More recently we have addressed the *nonlinear Schroedinger equation* in order to investigate the physical behavior of "rogue" waves and their IST spectral structure. Indeed it has been discovered that the dynamics of wave trains may be spectrally (in the sense of *nonlinear* (IST) Fourier analysis) decomposed into two types of components: (1) near linear Stokes waves and (2) unstable components which may be described as "rogue" waves.

WORK COMPLETED

The numerical results for the dynamics of rogue waves in random sea states have been completed as described below.

RESULTS

The single most important result of the present year's research has been the *numerical discovery of rogue dynamics in simulations of random wave trains*. Freak waves are extraordinarily *large water waves whose heights exceed by a factor of 2.2 the significant wave height* of a measured wave train. The mechanism of freak wave generation has become an issue of principal interest due to their potentially devastating effects on ocean going Naval and merchant ships and offshore structures of all types.

Indeed we have conduced simulations, which begin with initial conditions computed by ordinary linear Fourier analysis using the JONSWAP power spectrum and random Fourier phases. Initial conditions of this type are then numerically integrated forward in space and time using the nonlinear Schroedinger equation or its higher order extensions. We find that the random waves evolve nearly as one would normally expect in *linear simulations* of random waves, with the important exception that *every once in a while a large rogue wave rises up out of the random background sea state*.

In Figure 1, we show three examples of the JONSWAP power spectrum for different values of the *enhancement parameter*, γ. The major finding we have found in the past years is that, as γ grows, together with the *Phillips parameter*, α, the nonlinearity becomes more important and the probability for the formation of freak waves increases. When $\gamma = 1$, the resultant spectrum coincides with the "equilibrium range" Pierson-Moskowitz power spectrum. For larger values of γ, the peak of the spectrum is enhanced, even by as much as a factor of 10 (see Figure 1 for examples with $\gamma = 5$, $\gamma = 10$).

In order to address the particular circumstances that produce rogue waves, we have conducted a large body of numerical simulations in which we prepared initial conditions with the JONSWAP power spectrum for a chosen values of α γ and uniformly distributed random phases. Two examples are now discussed for ($\alpha = 0.0081$) $\gamma = 1$ and $\gamma = 10$. In Figure 2 is a space/time contour graph of the evolution for a typical case in which a rogue wave rises up out of a random wave sea state with significant wave height of 8 meters and a 20 meter rogue wave arises up out of the background (see Figure 3 also).

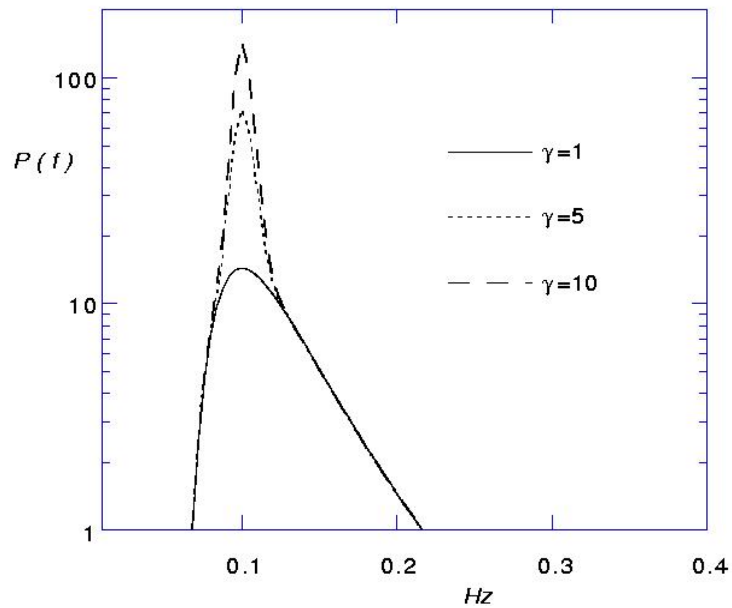

Figure 1. The JONSWAP Power Spectrum for Ocean Waves with Different Values of the Spectral Enhancement Parameter γ

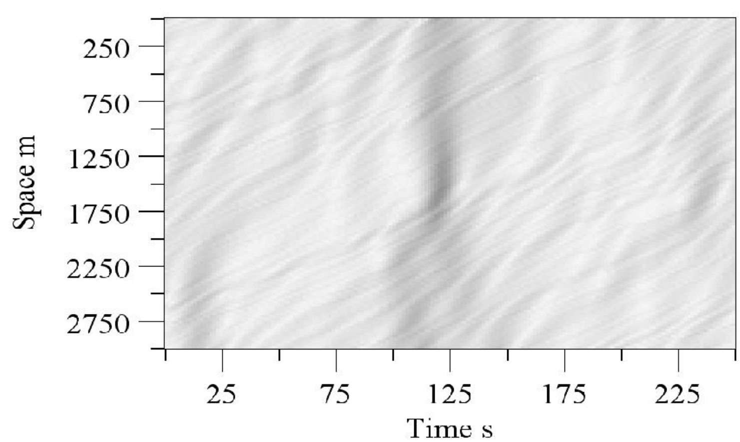

Figure 2. Space-time Evolution of a Rogue Wave in a Random Wave Train.

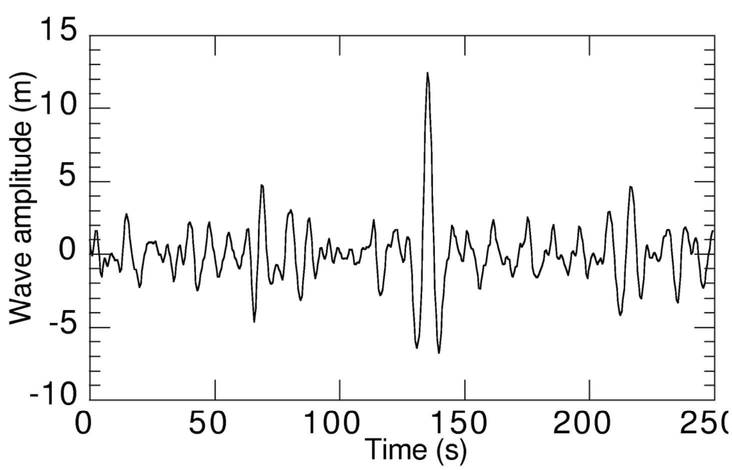

Figure 3. Time Series of a Random Wave Train Showing the Appearance of a Large Rogue Wave with Height 20 m Occurring at 140 Seconds.

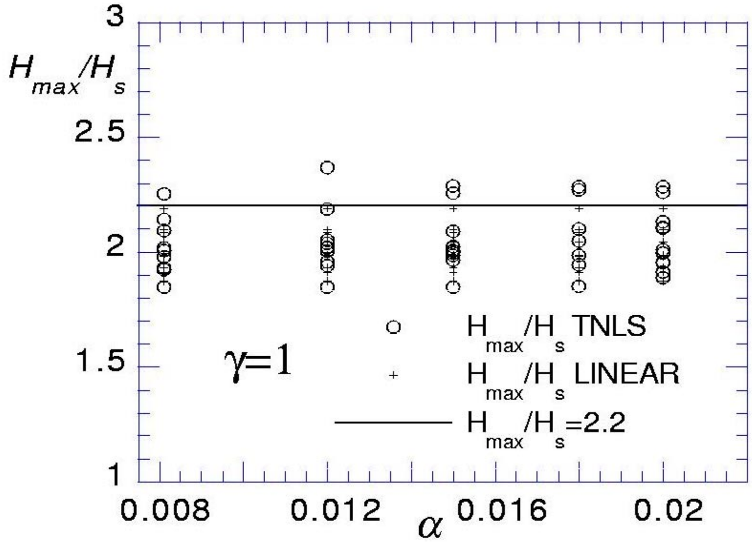

Figure 4. The Amplitude of Occurring Rogue Waves as a Function of the Parameter α for the Case $\gamma = 1$. Numerical Integrations of the NLS Equation and the Linearized NLS Equation are Shown. The Threshold at $2.2\ H_s$ is Also Shown.

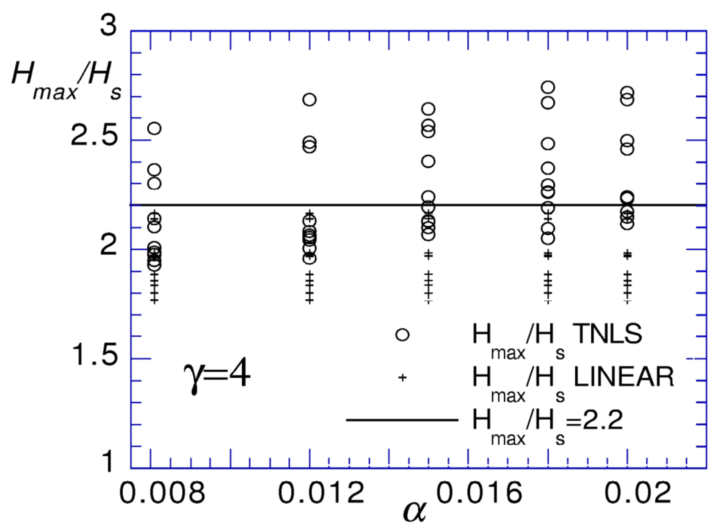

Figure 5. The Amplitude of Occurring Rogue Waves as a Function of the Parameter α for the Case $\gamma = 10$. Numerical Integrations of the NLS Equation and the Linearized NLS Equation are Shown. The Threshold at $2.2\ H_s$ is Also Shown.

To better understand how the occurrence of rogue waves depends on the parameters α, γ we have conduced a large number of simulations with varying values of the parameter; two of our results are shown in Figures 4 ($\gamma = 1$), 5 ($\gamma = 10$). Because of the small value of γ (= 1) in Figure 4, we might not ordinarily expect that rogue waves would occur. Indeed what we find is that the *probability* that rogue states occur is small as Figure 4 indicates (note values of wave height $\geq 2.2 H_S$). Increasing the value of gamma to $\gamma = 10$ we see in Figure 5 that there are many more rogues in the present case. The results of these simulations suggest that a probabilistic approach might be appropriate for describing the occurrence of rogue waves. Indeed we have found that this is quite difficult and that the selection of different sets of uniformly distributed random numbers in the initial conditions provides a *large variability in rogue wave behavior*. The use of IST to better understand the source of this variability therefore seems necessary and will be emphasized in future research.

IMPACT/APPLICATION

The impact of this research will occur in general for the nonlinear Fourier analysis of shallow and deep water wave trains, the analysis of internal wave trains and acoustic waves on the continental shelf, the design of floating surface and subsurface vessels, the fatigue life of tethered vehicles, etc.

TRANSITIONS

Collaborative work is now underway with Gene Tracy at the College of William & Mary (aspects of inverse scattering theory), Dave Kriebel at the U.S. Naval Academy and Don Resio at the U.S. Army Waterways Experiment Station.

RELATED PROJECTS

An intimate relationship between our results and other projects exists because the sea surface provides a major forcing input to many kinds of offshore activities, including the dynamics of floating and drilling vessels, barges, risers and tethered vehicles. The present work leads to a nonlinear representation of the sea surface forcing and vessel response.

REFERENCES

A.R. Osborne, 2000. Nonlinear Ocean Waves and the Inverse Scattering Transform, to appear in *Scattering*, edited by R. Pike and P. Sabatier, Academic Press.

A.R. Osborne, 2000. Nonlinear instability analysis of deep water wave trains: Computation of maximum heights of rogue waves, *Proceedings of OMAE 2000*, 19th International Conference on Offshore Mechanics and Arctic Engineering, Feb. 14-17, New Orleans, Louisiana, USA.

A.R. Osborne, M. Onorato and M. Serio, 2000. The nonlinear dynamics of rogue waves and holes in deep-water gravity wave trains, to appear in *Phys. Lett. A*.

M. Onorato, A.R. Osborne, M. Serio and S. Bertone, 2000. Freak Waves in Random Oceanic Sea States, submitted.

A.R. Osborne, 2000. The random and deterministic dynamics of "rogue waves" in unidirectional, deep-water wave trains, to appear *Jour. Marine Structures*.

A.R. Osborne, 1999. Rogue Waves and Holes in the Sea, report for the Mobile Offshore Base Program, sponsored by the Office of Naval Research.

M. Onorato, A.R. Osborne and M. Serio, Nonlinear Dynamics of Rogue Waves, to appear in the *Proceedings of the 6th International Workshop on Wave Hindcasting and Forecasting*, Monterey, California.